## The SCIENCE of AUDIO <br> 

Subwoofers: Optimum Number and Locations by
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## Low Frequency Optimization Goals

## 1. Get lots of output

2. Get lots of output, with flat frequency response at a single listening seat
3. Get lots of output, with flat frequency response over a defined listening area

Intuition tells us that putting a large number of subwoofers at different locations in a room is likely to excite room modes in a more "balanced" manner, as compared to a single subwoofer. This idea has potential where there is not a single listening location, but rather a listening area. In this case we look for consistency of acoustical response with in this area. One way to approach this problem is to excite all modes evenly. Another approach is to excite as few modes as possible. Using simulations and measurements we have made an investigation to determine if using a large number of subwoofers is advantageous, and in particular what configurations give the best results. Several interesting and surprising results were uncovered along the way.

Get lots of output. The high level of acoustical output we take for granted in today's subwoofers was not always a given. The "get lots of output at any price" approach is certainly not dead (witness nightclubs and many car stereo systems).

Get lots of output, with flat frequency response at a single listening seat. With 2-channel stereo systems and a single listener, it is natural to equalize for flat acoustical response of the combined subwoofer-room system. In fact, the interaction of room and loudspeaker is often the dominant factor in the perceived timbre [1]. This is especially true at lower frequencies.
Get lots of output, with flat frequency response over a defined listening area. With multichannel entertainment systems, there is more than one seat, so the acoustical response will be different at each seat, and the problem becomes more complex. Note that multichannel entertainment systems with large seating areas have been around for a long time, in the form of cinemas. It is when we see these systems reproduced in small rooms that low frequency modes start to become an issue.

With the advent of consumer multichannel systems, there is a potential for a large number of low frequency devices in a relatively small room to be operating simultaneously. This could occur either by running the mains and surround full range, as some advocate or by having multiple LFE's. From an intuitive standpoint, putting a large number of subwoofers at different locations in a room might seem likely to excite room modes in a more "balanced" manner, as compared to a single subwoofer. This idea has particular appeal where there is not a single listening location, but rather a listening area. Typical approaches to this problem have involved exciting all modes evenly, or trying not to excite modes at all. There is not much agreement on whether this is possible, how many subwoofers are required, and where best to place them.

- The locations of the subs might be selected to optimize the modal response of the room.


## The Crux of the Matter

- It is possible, and perhaps desirable to have multiple subwoofers operating simultaneously and coherently.


## Existing Research

- Mostly involves single subwoofers and/or single listening locations.
- Not a lot of work involving positional equalization with multiple sources and listening locations has been done.


## Questions Posed

- Is there a correlation between number of subwoofers and desirability of frequency response?
- How many subwoofers are enough?
- What is the optimal placement?


## Limiting Assumptions

- Rectangular room, subs located along walls.
- System is equalized
- Subwoofers driven coherently
- Maximizing LF output is secondary goal
- We are interested in acoustical response in a seating area, not at one particular seat

There is a substantial body of research on optimization of loudspeaker-room interaction at low frequencies, sometimes referred to as positional equalization [2], [3]. See [4] for an overview. Much of this work is directed toward optimization for a single listening position, and a single subwoofer. Of more interest to the present study are [5], [6], and [7] which assume a listening area. None of these studies look at more than a couple of subwoofers with more than a couple of configurations. Currently, there is much interest in multipoint equalization via complex matrix inversions or adaptive filters, as for example in [8]. Some of this work also considers positional equalization, however it is not the primary focus.
These are the specific questions we address in this paper.

Some corollary questions must be addressed:

- Is computer optimization feasible?
- What metric is appropriate to quantify performance of various subwoofer configurations?

These are assumptions we made to simplify our investigation a bit. Unequalized systems are briefly considered as well, in Investigation 8.

A large percentage of listening rooms are rectangular, but for those that are not, conclusions reached here are not valid.

Even with the above assumptions, there are many variables left to consider, making a complete analytical treatment difficult. This study is somewhat empirical in nature, and is broken up into a number of smaller investigations. It is hoped that the cumulative results of these investigations, taken together, will provide satisfying answers to our questions.

## Investigations

1. Large numbers of subwoofers
2. Rule of thumb placement
3. Brute force optimization
4. "Typical" subwoofer locations
5. Different room dimensions
6. Subwoofers in real rooms
7. "Typical" listening locations
8. Effect of equalization


## Possible Subwoofer Optimization Goals

- Minimize variation of frequency response from seat to seat
- Minimize variation of overall frequency response within the seating area
- Maximize low frequency output
- Optimize "imaging" or other spatial attributes of subwoofers

These are the investigations we made in this study.

But before discussing the investigations, some background...

The first goal for low frequency optimization should be to minimize variation of frequency response from one location to the next within the seating area. Assuming the room and listening area dimensions are fixed, the only way to do this is optimization of the number and location of subwoofers within the room. The second goal should be to minimize variation of overall frequency response within the seating area, i.e. take the spatial average of frequency responses at all seats and equalize this average frequency response flat.

If the first goal is not addressed, subsequent efforts at equalization will be of questionable value. If the frequency response from one seat to the next varies too greatly, equalizing the sound flat at one seat will simply make the frequency response less flat at another seat. We cannot have our cake and eat it too.

A secondary goal should be to maximize low frequency output of the subwoofers. Other, less quantifiable factors in subwoofer optimization are: practicality of subwoofer locations, number of subwoofers (cost), and possible spatial attributes [9].

## Subwoofer Optimization Goals in this Study

## - Minimize variation of frequency <br> Primary consideration

 response from seat to seat- Minimize variation of overall frequency response within the seating area

Maximize low frequency output

- Optimize "imaging" or other spatial attributes of subwoofers

Secondary consideration
Not considered for the most part (assume spatial average is equalized flat)

Not considered. Subs not localizable below 80 Hz anyway!


In this investigation, bass management is assumed, i.e. there is one and only one audio low frequency channel, which is sent to all subwoofers. There are many advantages to bass management and only questionable advantages to using multiple bass channels ("stereo" bass etc.). There is much debate on this subject. Much of it has to do with the implementation of the bass management headroom, mixing of LFE etc. This is another subject entirely, and is not considered here.

Some have argued that there is a subtle envelopment which can occur at low frequencies when out of phase material is reproduced on the left and right side of the listener [9]. This has not been shown conclusively, and in any case is likely outweighed by the more immediate advantages of bass management.

Some will also argue that bass below 80 Hz IS localizable. This is often the result of

Port noise in ported subwoofers which are poorly designed and/or overdriven

Non-linear distortion in subwoofers which are poorly designed and/or overdriven

Visual cues or knowledge of the subwoofer location, which in the absence of actual audible cues cause localization

Effective acoustical crossover frequencies which are actually higher than 80 Hz . Just because the electronic crossover is at 80 Hz does not mean that the actual crossover as measured in the room is at 80 Hz , especially if the crossover is low order.

## Multiple Subwoofers, Single Low Frequency Channel

Two big advantages of bass management:

- Allows mains and surrounds to be smaller/more efficient (they don't have to reproduce bass).
- Consistent frequency response. Sending low frequency signals to multiple loudspeakers results in multiple, and often widely diverging frequency responses!

A Tale of Five Subwoofers


The "classic" monitoring arrangement in a $20^{\prime} \times 24^{\prime}$ room

## A Tale of Five Subwoofers

Five measurements of the five loudspeakers


Note wide variance from one loudspeaker to the next at low frequency. THIS CAN BE AUDIBLE...

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Some have argued that there is a subtle envelopment at low frequencies which is enhanced when out of phase material is reproduced at the left and right side of the listener [9]. This has not been shown conclusively, and in any case this subtle effect is likely outweighed by the more immediate advantages of bass management.

## Signal Panned to Each Speaker.

Note that the sound from the different subwoofer locations might not sound as different as the frequency responses suggest visually, the difference may certainly be audible. Would we allow this degree of variation in any loudspeaker or amplifier if we could avoid it? The answer is an emphatic NO.

## Modal Behavior is Complicated!

- Modes are complex - literally!
- Modes overlap. They interact!
- "Eyeballing" expected room responses from generalized standing wave plots is tricky.

The modal behavior sound in rectangular rooms is well described in the literature, as in [10], [11], and [12], however it is often oversimplified or misunderstood. There are some aspects of room modes which make "eyeballing" expected room responses from generalized standing wave plots risky. Modal resonances have a finite bandwidth, that is to say they do not occur only at one discrete frequency. This means that adjacent modes will overlap to some degree (quite a bit if the room has 2 or more similar dimensions). If you further consider that modal response is complex, i.e. has a phase component, it can be seen that the interaction of multiple adjacent modes over a range of frequencies is complicated. When you have a defined seating area rather than a single seat, things can really get convoluted.

Due to the complexity of room modal response and the desire to investigate a large number of subwoofer/room configurations, an accurate room model is needed. Fortunately, modeling a rectangular room is relatively straightforward. A room modeling program was written using Matlab ${ }^{1}$ to model various configurations. This is based on the well-known closed form solution of the wave equation in a rectangular enclosure [13].

The actual response will be the sum of the reverberant sound $\left(P_{r}\right)$ and the direct sound from the subwoofer.

[^0]
## Simulations using Matlab ${ }^{\circledR}$

## These may look familiar:


$1^{\text {st }}$ AXIAL MODE

$2^{\text {nd }}$ AXIAL MODE

## Manipulating Room Modes

- Place speakers at or near nulls
- Place listeners at or near nulls
- Cancel modes


## Simulations using Matlab ${ }^{\circledR}$

No sound at nulls


## Simulations using Matlab ${ }^{\circledR}$

## No coupling at nulls = No excitation


$1^{\text {st }}$ AXIAL MODE

$2^{\text {nd }}$ AXIAL MODE $\quad{ }^{20}$

These are some ways which room modes can be mitigated.

George doesn't hear anything at the nulls!

Loudspeakers located at a null point of a given mode will not couple to that mode, resulting in no excitation of that mode.

## Simulations using Matlab ${ }^{\circledR}$

## Cancellation


$1^{\text {st }}$ AXIAL MODE

$2^{\text {nd }}$ AXIAL MODE

## Metrics

- Some type of metric is required to characterize one frequency response as better or more desirable than another.
- The standard deviation of the frequency response at each seat from the spatial average of all seats is one such metric.


## Test room with Listening Area



## Simulations using Matlab ${ }^{\circledR}$

Example of predicted
Frequency response(s)


## Simulations using Matlab ${ }^{\circledR}$

Predicted TOTAL SPL


## Statistical metrics based on equalized frequency responses

## Example:



A typical room simulation looks like this. There are 16 individual predicted frequency responses, as well as the calculated average (heavy black line). This is a spatial average. The red line show the "direct" sound from the subwoofers. Modal classification is also shown, i.e. axial, tangential, and oblique.

The direct plus modal (i.e. reverberant) sound is the total soundfield, and this is what we are interested in.

If we equalize so that the spatial average is flat, the 16 frequency responses look like this. We can then calculate the standard deviation of the (equalized) frequency response at each seat. Averaging the standard deviations at all 16 seats gives a single number metric, the Std.

In addition to the Std metric, two other metrics were used in this investigation: the difference between the absolute maximum and absolute minimum frequency response (MaxMin ), and the difference between the maximum and the average of the frequency response (Max-Ave). These metrics are more likely to reflect the effect of a single prominent frequency response feature on the subjective sound.

$n=$ number of active subwoofers.

Investigations: General Procedures


## Investigation 1: Really Large Numbers of Subwoofers

Can a sufficiently large number of subwoofers cancel out all room modes?

In addition to the frequency based metrics, it would be nice to know how well a subwoofer configuration performs with respect to overall low frequency output. The LF factor metric is simply the sum of the energy over the bandwidth of interest (20-80 Hz here) produced by a given configuration, assuming a correction factor to normalize for the number of subwoofers: $20 \log _{10} n$.

For all metrics used in this study, calculated values at each seat are averaged over the entire seating area grid, to give a single number.

Most of the experiments were carried out in a $20^{\prime} \times 24^{\prime} \times 9^{\prime}$ virtual test room. These dimensions match the dimensions of the physical listening room chosen for Investigation 6, where actual room measurements are used instead of simulations. Though most simulations are based on a $20^{\prime} \mathrm{x}$ 24 ' room, the effect of varying room dimensions is addressed in Investigation 5.

The purpose of this experiment is to test the idea that placing a large number of subwoofers in a room would in theory virtually eliminate any modal excitation. Each subwoofer excites a given mode according to its coupling function (the same curves as in Fig. 2-1), which in turn depends on its location in the room. For any given mode, placing subwoofers at locations of equal coupling magnitude and opposite phase will result in no excitation of the mode.

Using a large enough number of subwoofers distributed around the room should theoretically result in approximately equal positive and negative excitation of all possible modes.


## Investigation 1: Really Large

 Numbers of Subwoofers

Investigation 1: Really Large Numbers of Subwoofers


1st AXIAL MODE


PHASE/MAGNITUDE OF EXCITATION ${ }_{32}$

Remember: modes are complex. They have a different magnitude and phase at different locations in the room. Different subwoofer locations and/or different listening locations result in different phases for the room response.

The magnitude and phase of multiple resonances can be represented as a phasor diagram.

This shows lots of subwoofers with varying magnitudes and phases due to different (random) locations.



## Investigation 1: Really Large Numbers of Subwoofers

- 50 subwoofers randomly located in test room:




## Investigation 1: Really Large Numbers of Subwoofers

- 5000 subwoofers randomly located in test room:



## Investigation 1: Really Large Numbers of Subwoofers

Can a sufficiently large number of subwoofers cancel out all room modes?

## Theoretically yes

Practically NO

## Investigation 2: Rule of Thumb Subwoofer Placement

- Lets face it, its fun to think up cool subwoofer configurations!


- Subwoofers (4) at $1 / 4$ of room dimensions


For frequencies below 80 Hz , a virtually flat and uniform set of frequency responses could be achieved with 1 band of parametric equalization!

Knowing the basic spatial distribution of energy for each order and type of mode, it is tempting to come up with simple rule-of-thumb subwoofer configurations. For example, put one subwoofer in the corner to excite all the modes, or put in opposite corners to cancel out odd order modes. Most of these rules of thumb are based on axial modes only, ignoring the effect of tangential modes (oblique modes can generally be ignored). In addition, room response between modes is ignored.

This configuration should result in cancellation of all odd order axial modes, and the cancellation of the first even order axial mode (where the subwoofers are at a null).

This can be verified by looking at generalized standing wave plots like those on pages 17-21.

Note that floor/ceiling axial modes would not be cancelled, however these modes are much less important. Floor/ceiling axial modes do not cause variation in frequency response over the seating are, assuming ear height does not vary.

In this case, rule of thumb subwoofer placement worked well.


| Std | Max-Min | Max-Ave | LF |
| :--- | :--- | :--- | :--- |
| 1.3 | 4.6 | 2.3 | 19.7 |


B

Using the "eyeball" method of predicting modal response, one would be tempted to say that the configuration in A should result in cancellation of all odd order axial modes (ignoring floor-ceiling axial modes). The configuration in B should result in cancellation of all odd order axial modes in one direction and will not excite any odd order modes in the other direction, since it is at a node. Both subwoofer configurations above should result in no excitation of odd order axial modes.

The calculated metrics show very different results, with the configuration in A having a much higher seat to seat variation.

The calculated metrics show very different results, with the configuration in A having a much higher seat to seat variation than $B$.

## Investigation 2: Rule of Thumb Subwoofer Placement

- May be useful
- Results often not intuitive
- Tangential modes may be significant
- What happens in-between modes
- Frequency response is complex


## Investigation 3: Brute Force Computer Optimization

- Let the computer try all possible combinations, and pick the best
- Can be time consuming, not practical for more than a few subs
- Do in Matlab to have control over methodology

Investigation 3: Brute Force Computer Optimization



Rather than picking subwoofer locations using analytical or rule of thumb methods, is it certainly possible to use brute force to find the best number and locations. There are a number of commercially available programs which do this [CARA], [Room Optimizer]. Unfortunately, neither of these calculates metrics in the same manner as was desired for this study. CARA uses ray tracing, which while allowing optimization of non-rectangular rooms, is either not accurate (low order ray tracing) or too time consuming (high order ray tracing). Room Optimizer only optimizes for one seat, not a seating area. Even with fast computers, optimization can be extremely time consuming.
In order to do optimization for small numbers of subwoofers using the methodology in this investigation, an optimization routine was written in Matlab.

The next four figures show results of optimization of 1 to 4 subwoofers in the test room. Locations of subwoofers were constrained to be along the walls, at intervals of 2 feet. Optimization is based on Std only, i.e. the configuration with the lowest std is considered optimum.


Investigation 3: Brute Force Computer Optimization


## Investigation 3: Brute Force Computer Optimization

- Best results found with four subwoofers
- Two subwoofers almost as good, with better low frequency support
- Symmetrical locations seem best


## Investigation 4: Practical Subwoofer Locations

- What about "typical" subwoofer locations?



This figure shows subwoofer configurations chosen for the experiment.


This figure shows subwoofer configurations chosen for the experiment.


Results of Investigation 4.

There is no obvious trend towards better metrics when more subwoofers are used


The LF generally goes down for larger numbers of subwoofers, possibly due to having more locations away from the corners.


## Investigation 4: Practical Subwoofer Locations

- Best 3 configurations:




## Investigation 6: Real Rooms

- Do predicted results match measured results in a real room?



## Investigation 6: Real Rooms



Subwoofer locations chosen for the experiment are shown here. Due to several factors, including a sizeable rear projection television located at the front center of the room, and a limited number of available subwoofers, the configurations are slightly different than those modeled in Investigation 4.

Note that this set of configurations (21-37) was only used for Investigation 6, i.e. real rooms. For the rest of this paper, configurations 1-20 are used.

## Investigation 6: Real Rooms



31


36


32


37


33


34


35


The test room used in the computer modeling is based on a real $20^{\prime} \times 24^{\prime} \times 9^{\prime}$ listening room. This room is of standard stud/sheetrock construction. The only source of significant absorption at low frequencies is the sheetrock walls, and a large sofa. The room ,as set up for these tests, was fairly live ( $500 \mathrm{~Hz} \mathrm{RT} 60=.85$ ). This room is known to have unevenly distributed complex boundary absorption, due most likely to a door which reduces the structural stiffness of one wall. The result is that the nodes and antinodes along one dimension of the room are "skewed" to one side, and the measured eigenfrequencies of many modes do not exactly match the calculated values. Note that this condition is not at all unusual in real rooms.

In-room measurements were made using a custom fabricated grid with AKG-C98 microphones suspended at seated ear height.

## Investigation 6: Real Rooms

- Real versus measured results:



## Investigation 7: "Typical" Listening Locations

- What about:
- Seats moved back


66

## Investigation 7: "Typical" Listening Locations

- What about:



## Investigation 7: "Typical" Listening Locations

- Results using modified seating grid:



## In general, conclusions from Investigations 4-6 unchanged

## Investigation 8: Effect of Equalization

- Are results dependent on equalizing the system?



## Investigation 8: Effect of Equalization

- Results using no equalization:


It can be seen that there is less difference between "good" and "bad" seats, and that there is less low frequency support for some configurations compared to when the grid is centered and unweighted. Two and four subwoofers at the wall midpoints are still the best configurations overall. Four subwoofers in the corners does not seem as advantageous as when the grid was centered. There is still little or no advantage to using a large number of subwoofers. LF factor still goes down for higher numbers of subwoofers.

The Std, Max-ave and Max-min are significantly higher overall as compared to the centered unweighted seating grid.

Up to this point, it has been assumed that the playback system would be equalized such that the spatial average over the seating area is perfectly flat, as seen on the left side of this figure. This is not as unreasonable as it seems, given the fact the spatially averaged response is smoother than the frequency response at individual seats, and therefore easier to equalize flat.

It is also of interest to look at the case where there is no equalization at all. This is shown on the right side of this figure. Using subwoofer configurations 1-20, an experiment was conducted to test the effect of assuming no equalization. The calculated metrics then include not just seat to seat variation, but variation of the overall spatial average.

Not surprisingly, the Std, Max-min and Max-ave are much higher when no equalization is included. The general shape of the curves is still the same as for the case of no equalization. One subwoofer at each wall midpoint (\#11) is still the best configuration. Configurations \#19 and \#20 are almost as good, but why spend the money? There is no change in the LF factor when no equalization is used.

## Investigation 8: Effect of Equalization

- Are results dependent on equalizing the system?

Some effect, but does not change overall conclusions

## THE FOLLOWING <br> CONCLUSIONS ARE BASED ON THE ASSUMPTION OF A RECTANGULAR ROOM AND A SEATING AREA IN THE CENTER OR CENTER-REAR OF THE ROOM

## CONCLUSIONS

- Is there a correlation between number of subwoofers and desirability of frequency response?

For practical number of subwoofers, there appears to be no obvious correlation. There is certainly no justification for using more than four.

Using very large numbers of subwoofers would result in cancellation of room modes. For practical numbers of subwoofers, there appears to be no obvious correlation. When you consider the additional expense of using more subwoofers, there is certainly no justification for using more than four. To the contrary, it was observed that the LF factor actually went down for larger numbers of subwoofers

Using very large numbers of subwoofers would result in cancellation of room modes. For practical numbers of subwoofers, there appears to be no obvious correlation. When you consider the additional expense of using more subwoofers, there is certainly no justification for using more than four. To the contrary, it was observed that the LF factor actually went down for larger numbers of subwoofers

## CONCLUSIONS

- How many subwoofers are enough?

Four subwoofers are enough to get the best results of any configuration tried. Two subwoofers is very nearly as good and has very good low frequency support as well.

## CONCLUSIONS

- What is the optimal placement?


## Best configurations are:



## FUTURE WORK

Further work needs to be done to verify the adequacy of the metrics used here, vis á vis their correlation to actual listener preference. It is likely that there are better metrics which could be developed, though this is not a simple task. This is the subject of current investigation.
The combination of the positional approach described here and a multiple channel multiple receiver equalization technique ([16] [17]) should also be pursued.

One subwoofer at each wall midpoint is the best in terms of Std, Max-ave and Max-min but does not support low frequencies particularly well. Two subwoofers, at opposing wall midpoints, performs very nearly as well as four at the midpoints and gives a much better LF factor. One subwoofer in each corner also has good low frequency support, but does not perform quite as well as one subwoofer at each wall midpoint, in terms of Std, Max-ave and Max-min. If cost and aesthetics are considered, subwoofers at $\underline{\underline{2}}$ wall midpoints is preferred.

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& \text { http://www.aes.org/publications/preprints } \\
& \text { /lists/112.cfm }
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[^0]:    ${ }^{1}$ Mathworks Inc. www.mathworks.com.

